shown for a channel cross section 2 m from two diametrally positioned direct-flow nozzles, orthogonal to the flow. The channel diameter is 1 cm, the velocity of disperse-phase injection is the same for both nozzles (35 m/sec); all the other parameters of the nozzles and the gas flow are "standard." The dashed lines in Fig. 4b show the distributions given by each nozzle and the continuous curve shows the total mass distribution function of drops over the channel cross section. It follows from the resulting calculations that the correct choice of parameters of the injection system may ensure a very homogeneous distribution of the disperse phase even with a small number of nozzles.

LITERATURE CITED

1. V. A. Raushenbakh, S. A. Belyi, I. V. Bespalov, et al., Physical Principles of the Working Process in Combustion Chambers of Air-Reactive Motors [in Russian], Mashinostroenie, Moscow (1964).
2. Ya. S. Zholudov, "Choosing the effective trajectory of particle injection by the attachment in the combustion chamber of a magnetohydrodynamic generator," in: Engineering Problems of Magnetic Hydrodynamics [in Russian], ITTF, Kiev (1978), pp. 83-88.
3. I. L. Mostinskii, D. I. Lamden, D. K. Burenkov, and A. V. Zagorodnikh, "Cooling of high-temperature gas in a scrubber," Prom. Teplotekh., 2, No. 3, 97-103 (1980).
4. A. N. Shurygin, Design of Nozzles for Liquid Spraying [in Russian], MEI, Moscow (1964).
5. D. I. Lamden and I. L. Mostinskii, "Evaporation of solvent from a drop of solution moving in a hot gas," Teplofiz. Vys. Temp., 14, No. 4, 804-813 (1976).

PROPAGATION OF SMALL DISTURBANCES IN CONCENTRATED DISPERSED SYSTEMS

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The mechanism of elastic pressure waves in concentrated dispersed systems is discussed. It is shown that the continuously relaxing medium model is valid for describing acoustic effects in a fluidized vibrating layer.

Concentrated dispersed systems of the "fluidized layer" (FL) type are characterized by the essential nonstationarity of all hydrodynamic processes due to the nonlinear properties in the particle bulk concentration. The propagation laws of dynamic disturbances play an important role. The dynamic FL characteristics earlier considered were usually related to propagation of comparatively slow plastic isolated waves during spontaneous or induced change of flow of a fluidized agent [1, 2]. Their appearance was related to quasielastic relaxation processes due to the nonlinear dependence of the aerodynamic particle resistance on their spacing density, earlier found by Roy [3]. In this case the analysis included only low layers, with pressure waves propagating along them practically instantaneously [4, 5]. One of the most interesting effects, explained within the concepts of incompressibility of the fluidized agent in an FL, is the effect of ordered oscillations of the gas pressure and of the dispersed phase density (in the form of self-oscillations in boiling [6] and induced oscillations in pulsating [1] and vibrating-boiling [5] layers), characterizing the law of expansion cycle, precipitation at each period of oscillation, occurring with a completely determined "zero-order" frequency $f_0 \sim \sqrt{g/H_0}$.

Imposing on an FL induced oscillations with a frequency larger than the zero-order frequency ($f_B > f_0$), the action of the relaxation oscillations is restricted by the surface and bottom portions of the layer, and their contribution to the formation of the internal portions of the FL is diminished. At the same time the passage of a pressure wave through a high FL is compatible with an oscillation period T_B. Under these conditions one must expect the appearance of gas compressibility (elasticity), which would lead to resonance effects of higher than "zero" order.

At present the model of interacting, mutually penetrating continua is most widely used to describe the behavior of dispersed systems. This model is valid when the characteristic

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scale of the effect exceeds the size of the microinhomogeneity of the dispersed medium. To this belongs propagation of elastic oscillations in a dispersed medium with a wavelength significantly exceeding the particle diameter and the mean distance between them. For tenuous fluidized beds and "gas" or moist vapor type media the presence of a mechanism of propagation of elastic oscillations in the form of acoustic waves was proved (see, for example, [7]), while an analytic description of sound propagation in these media results from a justified assumption on particle interaction with each other through the supporting phase only. This a priori simplification is too crude in concentrated dispersed media. Therefore, several investigators have doubted the very possibility of existence of elastic waves in concentrated dispersed systems of the FL type, where the mean distance between the centers of the dispersed particles is on the order of their diameter [2]. This doubt is fully justified due to the nonlinearity of the object and the presence of significant dissipative effects. Only experiments can indicate the presence of elastic properties in FLs, implying the possibility of propagation of elastic oscillations in concentrated dispersed systems, considered as homogeneous media with effective characteristics.

The problem of studying the dynamic properties of FL is complicated by the difficulty of creating a stable, macroscopically uniform fluid (without bubbles) with properties assigned ahead of time. At this stage the most suitable is the layer of findly dispersed particles, oscillating as a quasifluid (VL). For certain combinations of oscillation parameters (f_B and A_B), properties of the dispersing medium, and its elements (H, ρ_1, d_1, μ_0, p_0) there occur expansion and fluidization of solid particles in the oscillating bulk of the gas, accompanied by relatively slow secondary phase transformations. In this case the expansion is maintained at a high level, and the aggregate of solid particles never separates at the bottom of the cavity. This is proved by the well-known fact of immobility of the upper boundaries of the expanding VL for practically nonselecting motion (from the bottom) of the lower parts of the layer with the relation $\Delta H/A_B \sim 10^1$.*

Thus, the separation pulsation (occurring with frequency f_B) does not exceed the mean by more than 5-7%, exceeding the separation of the precipitated layer by 30-50%. Taking into account that in a slightly dispersed fluidized filling the pulsations of gas pressure p_0 and of particles p_1 are of the same order (for example, for maximum expansion $p_0 > 2p_1$ [10, p. 186]), we reach a conclusion of continuous stress redistribution between the gas and the suspended particles and the possibility of "freezing" of solid particles in the variable continuous phase.† It is natural to expect that momentum transfer from the vibrating bottom will be realized (to a large extent) by elastic pressure sound waves.

We discuss a possible mechanism of sound wave propagation in a primarily expanding dispersed mixture. Clearly, in a fluidized gas the speed of sound has the value a_0, depending on its temperature only. Let a cluster of particles, suspended in the gas, border with a pure gas, in which sound waves propagate with speed a_0 and angular frequency ω. If the wavelength corresponding to this frequency is comparable and larger than the mean distance between particles, the latter can be considered as a bulk diffraction lattice, where due to diffraction and interference, the incident sound wave will be almost completely absorbed. And since particles in the gas are not fixed, the incident wave reduces to oscillating motion of the external gas layer and particles, which becomes in turn an emitter of acoustic waves, propagating into the depth of the particle cluster. The velocity of propagating waves can be estimated from the thermodynamic relation

$$a^2 = -v^2\left(\frac{\partial p}{\partial v}\right)_S = \left(\frac{\partial p}{\partial \rho}\right)_S, \qquad (1)$$

whose validity for a dispersed medium is verified if the region of periodic medium compression and expansion (wavelength) is much larger than the distance between particles. The pressure p of the gas mixture and of the particles differs in most cases quite little from

*The topic here is not dynamic particle interaction with the bottom [8], but kinematic contact of small particles, "frozen" in the gas and not tearing away more than $0.2A_B$, which is easily observed, for example, by photography with stroboscopic illumination [9].
†A reliable indication of "soft" shockless fluidization under these conditions by the widely used fact in VL practice of the absence of noticeable wear and small particle cracking in optimal regimes of vibrating machines.

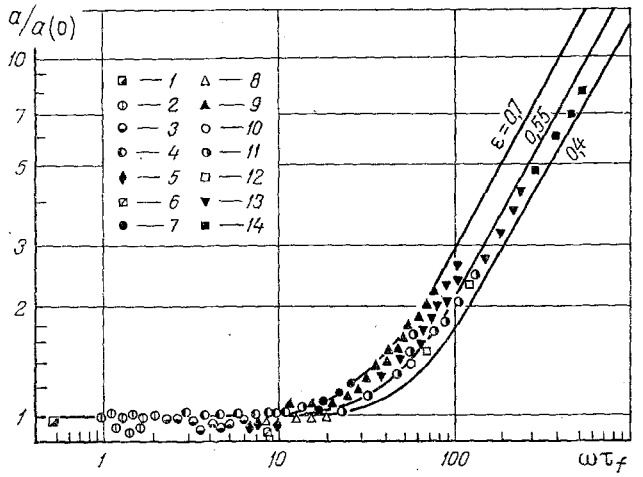

Fig. 1. The relative speed of sound as a function of the dimensionless frequency in the dispersed medium air—corundum: 1) d = 0.017 mm; 2) 0.028 [20]; 3) 0.046 [17, 19]; 4) 0.06; 5) 0.068 [17]; 6) 0.07 [29]; 7) 0.08; 8) 0.094 [20]; 9) 0.12; 10) 0.15 [18]; 11) 0.16; 12) 0.19 [20]; 13) 0.23; 14) 0.32; 15) 0.32 [27].

the pressure of a pure gas, and the specific volume of the mixture is calculated by the equation $v = 1/\rho$.

In a dispersed medium consider a portion of the front of a plane wave of unit area and width on the order of the wavelength. If the compression velocity (i.e., the oscillation frequency) is small, the particles will follow behind the gas. The number of particles N in the varying volume will then be a constant quantity equal to nV. From the constancy of N we find $dn = -(n/V)dV$. Assuming that the process of gas compression is adiabatic and neglecting its heat exchange with particles, we obtain, taking into account the constancy of the gas mixture in volume V,

$$\left(\frac{\partial p}{\partial v}\right)_S = -\frac{kp}{v(1-nv_1)}. \tag{2}$$

Thus, at low frequencies, according to (1) and (2), the speed of sound in a gas—solid particle mixture will be $a^2(0) = kp/\rho\varepsilon$. It is hence seen that the square of the speed of sound in a dispersed mixture is, for low frequencies, inversely proportional to the mixture density, which in a system of the FL type is quite high in comparison with the density of a pure gas. Thus, at low frequencies, when the particles at the wave front are completely entrained by the gas, their inertia lowers substantially the speed of sound.*

Consider now the case of high frequencies, when the particles can be assumed nonmobile during the passage of the wave front. In this case a constant quantity is no longer the number of particles in a varying volume V, but their number density n. Assuming primarily that the process of change of state of the gas is adiabatic, and neglecting heat exchange of the gas with particles, we find after noncomplicated calculations $a^2(\infty) = a_0^2/\varepsilon$.

*With account of heat exchange the adiabatic index k approaches 1. The exact expression for the equilibrium speed of sound, obtained in [13], is $a^2(0) = (p/\rho\varepsilon)\left(1 + \frac{p\varepsilon}{\rho c_v T}\right)$.

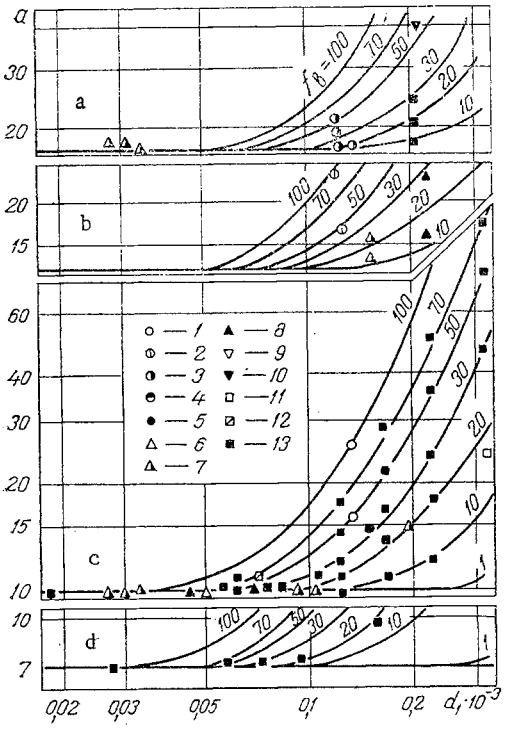

Fig. 2. The speed of sound a (m/sec) in a dispersed medium as a function of particle diameter d_1 (m) for f_B = idem (Hz) and ε = 0.55; a — ρ_1 = 1200 kg/m^3; b — 2400; c — 4000; d — 8000; 1) [9]; 2) [15]; 3) [16]; 4) [17]; 5) [18]; 6) [19]; 7) [20]; 8) [21]; 9) [22]; 10) [23]; 11) [27]; 12) [29]; 13) authors.

In the general case the propagation velocity of small perturbations in a dispersed medium depends substantially on the interphase interaction during passage of a wave front. If the particles are nondeformable, the interphase interaction is reduced to viscous particle friction and to interphase heat exchange. As shown by more general analysis, dissipative effects must necessarily lead to dispersion during propagation of small particles in a relaxing medium, i.e., to a frequency dependence of the phase velocity of sound and of the absorption coefficient. This problem, as applied to the systems of gas—solid particles, humid vapors, and liquid—vapor bubbles, was solved with various degrees of approximation by many investigators (see, for example, [11, 12]). In the present paper, to calculate the speed of sound in concentrated dispersed systems with temperature and relaxation rates, we basically use the results of [12-14], where by applying the relaxation apparatus of thermodynamics of irreversible processes, dynamic thermal and caloric equations of state of a dispersed system with temperature and relaxation rates were obtained:

$$-\frac{\delta\rho}{\rho^{\omega}} = \varepsilon\left[1 - \frac{\rho_1 c_1 (1-\varepsilon)}{\rho c_V}\frac{\tau_q D}{1+\tau_q D}\right]\frac{\delta T_0}{T_0} - \varepsilon\frac{p}{p}, \quad \delta u = (\rho c_V)^{\omega}\,\delta T_0 + \rho^{\omega}\,\delta(w_0^2/2), \tag{3}$$

$$(\rho c_V)^{\omega} = \rho c_V\left[1 - \frac{\rho_1 c_1 (1-\varepsilon)}{\rho c_V}\frac{\tau_q D}{1+\tau_q D}\right], \quad \rho^{\omega} = \rho\left[1 - \frac{\rho_1 (1-\varepsilon)}{\rho}\frac{\tau_f D}{1+\tau_f D}\right]. \tag{4}$$

For a monochromatic wave the operator D in these expressions is replaced by its eigenvalue $i\omega$, and the speed of sound in the mixture, according to (1), acquires the form

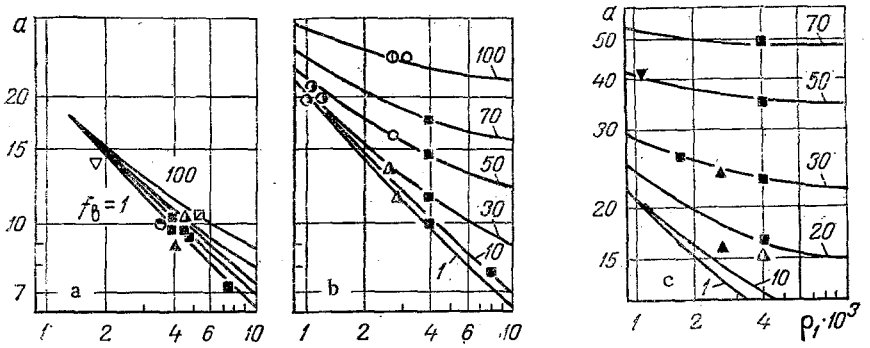

Fig. 3. The speed of sound a (m/sec) in a dispersed medium as a function of particle density ρ_1 (kg/m^3) for f_B = idem (Hz) and $\varepsilon = 0.55$; a — $d_1 = 0.06$ mm; b — 0.12; c — 0.22. The remaining notations are the same as in Fig. 2.

$$a^2(\omega) = \left(\frac{\partial p}{\partial \rho}\right)_S^{\omega} = \frac{p}{\rho}\frac{1+i\omega\tau_f}{1+i\omega\tau_f B_f}\left[\frac{1}{\varepsilon} + \frac{p}{\rho c_V T_0}\frac{1+i\omega\tau_q B_q}{1+i\omega\tau_q}\right], \tag{5}$$

where the relaxation times for solid spherical particles are expressed under the assumption of small Bi and Re.

In all practically important cases of fluidization $\rho_0 \ll \rho_1$, the voidage ε is not very close to unity, and $\tau_q \sim \tau_f$. Then, restricting ourselves to the linear approximation $a^2(\omega)$ in small B_f and B_q, it can be shown that in the whole frequency region the effect of interphase heat exchange on the speed of sound is smaller by approximately ρ_0/ρ_1 times than the relaxation rate; therefore, in what follows we neglect the temperature relaxation. In this approximation the real part of the square of the speed of sound is written in the form

$$a^2(\omega) = \frac{p}{\rho}\left(\frac{1}{\varepsilon} + \frac{p}{\rho c_V T_0}\right) - \frac{1+\omega^2\tau_f^2 B_f}{1+\omega^2\tau_f^2 B_f^2}. \tag{6}$$

In Fig. 1 we show by full lines the dimensionless form of the speed of sound in a mixture of air–corundum particles as a function of the dimensionless frequency $\omega\tau_f$ for various values of the voidage ε according to Eq. (6).

At low frequencies ($\omega\tau_f < 10$) the speed of sound in the mixture is comparatively low (~10 m/sec), which is related to the high inertia of concentrated mixture particles with high density ρ_1. With increasing frequency the particles do not succeed in following the gas oscillations, and become less mobile during the passage of the wave front. In this case the sound wave spends on the average part of the time in the gas with the true velocity a_0, and part of the time in the particle material (with significantly higher velocity). Therefore, at high frequencies the speed of sound in the mixture seems to be higher than the speed of sound in a pure gas. Figures 2 and 3 show the behavior of the speed of sound in a dispersed mixture as a function of particle diameter and density for given voidage and various frequencies. As is seen, the speed of sound in the mixture undergoes dispersion also with a change in particle size. The dispersion mechanism is such that for small particle sizes their inertial frequency is low, and with the passage of a wave front they move together with the gas particles, so that the sound wave propagates uniformly. For increasing particle size their inertia increases, which is equivalent to an increasing frequency. The higher the particle density, the smaller the size for which the speed of sound starts undergoing dispersion.

To test the theory several materials were used in the so-called optimal regimes in VLs of slightly dispersed particles, widely discussed in the literature [8-10, 16-24, 26, 28]. The analysis implies the practically simultaneous extremal change of the various VL parameters: dynamic (maximum pulsations of gas pressure and simultaneous minimal pressures of layer particles on the enclosing surface [8, 9, 15, 24, 25]), kinematic (minimum angle of particle separation from the bottom, maximum toss height and flight time of particles [8, 9, 24, 25]), structural (maximum loosening and expansion of filling, smoothing of surface layer, appearance of "suspended" or "spouting" regime), characteristics [8, 15, 16, 18, 20, 22-25], as well as transport coefficients (maximum heat-transfer coefficients, effective thermal conductivity, and mixing intensity [8, 16, 21, 24, 25]).

Analysis of the field of instantaneous gas pressure in the VL by the usual methods [8] has shown that a system of standing waves is generated in oscillatory fluidization in the optimal regime. The pressure amplitude in them is a periodic function of coordinates and is independent of time. Alternate nodes (p_0–min) and antinodes (p_0–max$_0$) of gas pressure occur across the layer height. The lower boundary always obeys the antinode condition, and on the upper boundary $p_0 \simeq 0$. The distance between adjacent antinodes approximately coincides with the distance between nodes. Pressure oscillations between two adjacent nodes occur in a single phase. During the passage of an instantaneous line of pressure p_0 through the nodes the oscillatory phase changes by 180°. The oscillation frequency of the gas pressure in all the layer cross sections usually coincides with the bottom oscillation frequency f_B. Therefore, the relation between the oscillation frequency and the VL height in all these optimal regimes obeys the well-known dependence for linear resonance of a column of elastic gas with density ρ in a tube open from above of height H during vibration of the bottom with velocity $A_B\omega \ll a$:

$$f_B = (2m - 1)\, a/4H.$$

We handled by this equation more than 250 regimes (of which around 100 were obtained by other authors), shown partially on Figs. 1-3, and comparison with direct measurements of velocities of pressure waves in air, carried out in the nonresonance [27] and resonance [28] regimes, shows convincingly that Eq. (6) describes satisfactorily the well-known experiment. This makes it possible to assume that the model used of elastic oscillation propagation in dispersed systems is fully adequate up to significant bulk concentrations of particles.

NOTATION

f and ω, frequency and angular frequency; g, free-fall acceleration; H_0 and H, height of the bulk and vibrating beds; f_B, T_B, and A_B, frequency, period, and amplitude of the forced oscillations; p, pressure; a, speed of sound; ρ, density; μ, dynamic viscosity; v, specific volume of the mixture; S, entropy; ε, voidage; V, volume of the distinguished wave front; N, number of particles in the distinguished wave front; n, particle number density; v_1, volume of an individual particle; k, index of the adiabat; c, heat capacity; T, temperature; τ_f and τ_q, respectively, times of velocity and temperature relaxation; $D \equiv d/dt$, a differential operator; u, specific internal energy; w, velocity; d, particle diameter; λ, thermal conductivity; $i = \sqrt{-1}$; B_f and B_q, complexes defined in (6), m = 1, 2, ... Subscripts: 0, pure gas, initial state; 1 partcle; , ω is the dynamic value, and $B_f = \rho_0\varepsilon/\rho$; $B_q = \rho_0\varepsilon c_{V_0}/\rho c_V$; $\tau_f = \rho_1 d^2/18\mu_0$; $\tau_q = \rho_1 c_1 d^2/12\lambda_0$.

LITERATURE CITED

1. M. H. J. Baird and A. J. Klein, "Spontaneous oscillation of gas-fluidized bed," Chem. Eng. Sci., 28, 1039-1048 (1973).
2. J. Verloop and P. M. Heertjes, "Shock waves as a criterion for transition from homogeneous to heterogeneous fluidization," Chem. Eng. Sci., 25, 825-832 (1970).
3. P. N. Roy and D. A. Henwood, "Pressure forces in the hydraulic model of a fluidized bed," in: Hydrodynamics and Mass Transfer in a Fluidized Bed [in Russian], Atomizdat, Moscow (1964), pp. 9-48.
4. J. W. Hibu, "Periodic phenomena connected with gas-solid fluidization," Chem. Ing. Technik, 39, 1125-1129 (1967).
5. Yu. A. Buevich and V. L. Gapontsev, "Fluidization of low granular layers," Inzh.-Fiz. Zh., 34, 394-403 (1978).
6. V. A. Borodulya, Yu. A. Buevich, and V. V. Zav'yalov, "Theory of relaxation self-oscillations of a granular layer cooled by a gas," Inzh.-Fiz. Zh., 30, 424-433 (1976).
7. V. D. Vorontsov, "Wave effects in a two-phase medium," Inzh.-Fiz. Zh., 21, 92-99 (1971).
8. A. F. Ryzhkov, "Pressure fluctuations of a gas in a vibrating layer," in: Industrial Furnaces with a Fluidized Bed [in Russian], A. P. Baskakov (ed.), Sverdlovsk (1973), pp. 12-17.
9. W. Kroll, "Über das Verhalten von Schuttgüt in lotrecht Schwingen den Gefassen," Forsch. aus dem Gebiete des Ingenieurwessens, 20, 2-5 (1954).
10. A. P. Baskakov, B. V. Berg, A. F. Ryzhkov, and N. F. Filippovskii, Processes of Heat and Mass Transfer in a Fluidized Bed [in Russian], Metallurgiya, Moscow (1978).
11. I. S. Radovskii, "Speed of sound in two-phase vapor–liquid systems," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 78-85 (1970).
12. E. M. Tolmachev, "Study of thermal and hydromechanical phase interactions in dispersed and fluidized systems," Author's Abstract of Candidate's Dissertation, Sverdlovsk (1972).

13. E. M. Tolmachev and N. I. Syromyatnikov, "Speed of sound in an equilibrium dispersed medium," Izv. Vyssh. Uchebn. Zaved., Energ., No. 4, 132-135 (1972).
14. E. M. Tolmachev, G. P. Yasnikov, and N. I. Syromyatnikov, "Equations of hydrodynamic flow of a dispersed medium in the relaxation approximation," in: Heat and Mass Transfer in Dispersed Systems [in Russian], Vol. 5, Part 1, Naukova Dumka (1972), pp. 24-28.
15. W. Kroll, "Fliesserscheinungen an Haufwerken in Schwingen den Gefässen," Chem. Ing. Technik, 27, 33-38 (1955).
16. É. M. Mitkevich, "Calcination of sodium bicarbonate in a fluidized bed," Zh. Prikl. Khim., No. 6, 1264-1277 (1960).
17. S. S. Zabrodskii, I. L. Zamnius, S. A. Malyukovich, and A. I. Tamarin, "Heat transfer by a layer of a thinly dispersed material, fluidized by a gas flow or by an applied vibration," Inzh.-Fiz. Zh., 14, 448-453 (1968).
18. I. L. Zamnius, A. I. Tamarin, and S. S. Zabrodskii, "Hydrodynamics and heat transfer of a fluidized bed," in: Heat and Mass Transfer [in Russian], Vol. 5, Nauka i Tekhnika, Minsk (1968), pp. 142-152.
19. I. I. Kal'tman and A. I. Tamarin, "Heat transfer between a fluidized bed of a dispersed material and a body cooled in it," Inzh.-Fiz. Zh., 16, 630-638 (1969).
20. I. L. Zamnius, "Heat transfer between a fluidized body and the surface," Author's Abstract of Candidate's Dissertation, Minsk (1970).
21. A. I. Tamarin and I. I. Kal'tman, "Effective temperature conductivity of a fluidized bed," Inzh.-Fiz. Zh., 20, 274-280 (1971).
22. I. L. Zamnius, "Hydrodynamic and structural features of a fluidized bed," in: Heat and Mass Transfer [in Russian], Vol. 5, Part 1, Naukova Dumka, Kiev (1972), pp. 158-163.
23. Sh. M. Bilik and N. N. Slyudikova, "Fluidization by vibration of powder-shaped polymer layers for coating," Inzh.-Fiz. Zh., 23, 664-672 (1972).
24. A. F. Ryzhkov, "Mechanism of vibrational action on a slightly dispersed filling in large instruments," Author's Abstract of Candidate's Dissertation, Sverdlovsk (1974).
25. A. F. Ryzhkov and A. P. Baskakov, "Motion of granular material in a high vibrating instrument," Inzh.-Fiz. Zh., 34, 1048-1053 (1978).
26. Yu. A. Buevich, A. F. Ryzhkov, and N. M. Kharisova, "Instability of a fluidized bed," Inzh.-Fiz. Zh., 37, 626-634 (1979).
27. V. S. Nosov, V. N. Korolev, and V. L. Gapontsev, "External heat transfer in dispersed systems with account of nonstationary processes," in: Heat and Mass Transfer [in Russian], Vol. 6, Part 1, ITMO, Minsk (1980).
28. A. F. Ryzhkov, A. P. Baskakov, A. S. Kolpakov, and A. V. Pavlichenko, "Optimization processes of heat and mass transfer in monodispersed and multifractional fluidized beds," in: Heat and Mass Transfer [in Russian], Vol. 6, Part 1, ITMO, Minsk (1980), pp. 104-108.
29. V. L. Gapontsev, "Mechanism of formation and heat exchange of a fluidized bed with a vertical surface immersed in it," Author's Abstract of Candidate's Dissertation, Sverdlovsk (1981).